Restarting Automata

with more than one restarting state

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Introduction and Preliminaries

Definition 1 (qRLWW-automaton)

$$M := (Q, \Sigma, \Gamma, \cent, \$, q_0, k, \delta, Q_R)$$

- *where Q, Σ, Γ, $\cent$, $\$$, q_0, k are defined as in ordinary* RLWW*-automata*
- $Q_R \subseteq Q$ *set of restarting states,* $q_0 \in Q_R$*, and*

$$\delta : Q \times \mathcal{P}C^{(k)} \to \mathfrak{P}((Q \times (\{\text{MVR}, \text{MVL}\} \cup \mathcal{P}C^{\leq(k-1)})) \cup (\{\text{Restart}\} \times Q_R) \cup \{\text{Accept}\})$$

Definition 2 (q-meta-instructions)

$$(p, R_1, u \to v, R_2, q) \text{ and } (p, R_1, \text{Accept}) \quad p, q \in Q_R$$

Proposition 1

$\mathcal{L}(q\text{RLWW}) \subseteq \text{NP} \cap \text{CSL} \quad \mathcal{L}(\det q\text{RLWW}) \subseteq \text{P} \cap \text{DCSL}$

Definition 3 (q_cRLWW-automaton)

Cardinality $|Q_R| \leq c$ *limited by constant* $c \in \mathbb{N}$.

Definition 4 ($\hat{q}_c$RLWW-automaton)

$q_0 \notin Q_R$ *and* $0 < |Q_R| \leq c$ *with constant* $c \in \mathbb{N}$.

Head of computation: $q_0 ¢ w \$ \vdash^c p ¢ \hat{w} \$ \ \cdots \qquad p \in Q_R$

Proposition 2

$\mathcal{L}(\text{RLWW}) = \mathcal{L}(q_1\text{RLWW}) \quad \mathcal{L}(\text{det-RW}) \subsetneq \mathcal{L}(\det \hat{q}_1\text{RW})$

Expressive Power $\det \hat{q}$RW-automata

Example 1 ($\{a^n b^n c^n \mid n \geq 0\} \in \mathcal{L}(\det \hat{q}_1 \text{RW})$)

$(q_0, ¢\, a^* b^* c^+, cccc\$ \rightarrow acc\$, q_1)$ $\quad$ $(q_0, ¢\, a^k b^k c^k \$, \text{Accept})$ *for* $k \in \{0, 1, 2, 3, 4$

$(q_1, ¢\, a^* b^* c^+, cca \rightarrow ac, q_1)$

$(q_1, ¢\, a^* b^+, bbca \rightarrow cb, q_1)$

$(q_1, ¢\, a^* b^+, bbcca \rightarrow abc, q_1)$

$(q_1, ¢\, a^* b^+, bba \rightarrow ab, q_1)$

$(q_1, ¢\, a^* b^+, bbcb \rightarrow cbb, q_1)$

$(q_1, ¢\, a^*, aabba \rightarrow cab, q_1)$

$(q_1, ¢\, a^*, aaabc \rightarrow ca, q_1)$

$(q_1, ¢\, a^*, aaca \rightarrow caa, q_1)$ $\quad$ $(q_1, ¢\,, c \rightarrow \epsilon, q_1)$

$(q_1, ¢\, a^* b^* c^+, cccc\$ \rightarrow acc\$, q_1)$ $\quad$ $(q_1, ¢\, a^k b^k c^k \$, \text{Accept})$ *for* $k \in \{1, 2, 3, 4\}$

Expressive Power $\det \hat{q}\mathsf{R}$-automata

Example 2 ($(\{a^n b^n c \mid n \geq 0\} \cup \{a^n b^{2n} d \mid n \geq 0\}) \in \mathcal{L}(\det \hat{q}_2\mathsf{R})$**)**

$$(q_0, \cent a^* b^*, c \longrightarrow \epsilon, q_c)$$
$$(q_0, \cent a^* b^*, d \longrightarrow \epsilon, q_d)$$

$$(q_c, \cent a^*, ab \longrightarrow \epsilon, q_c)$$
$$(q_c, \cent\$, \textsf{Accept})$$
$$(q_d, \cent a^*, abb \longrightarrow \epsilon, q_d)$$
$$(q_d, \cent\$, \textsf{Accept})$$

Corollary 1 $\mathcal{L}(\det \hat{q}\mathsf{R}) \not\subseteq \mathcal{L}(\mathsf{RW})$

Expressive Power $\det \hat{q}\mathsf{R}$-automata

Consider languages over $\Sigma = \{a, b, c_1, \dots, c_n\}$:

$$L_m = \bigcup_{1 \le i \le m} \{a^n b^{i \cdot n} c_i \mid n \ge 0\}$$

Conjecture 1 (hierarchy of $\det \hat{q}_i\mathsf{R}$-automata)

$$\mathcal{L}(\det \hat{q}_1\mathsf{R}) \subsetneq \mathcal{L}(\det \hat{q}_2\mathsf{R}) \subsetneq \dots \subsetneq \mathcal{L}(\det \hat{q}_i\mathsf{R}) \subsetneq \mathcal{L}(\det \hat{q}_{i+1}\mathsf{R})$$

Expressive Power $\det q\mathsf{R}$-automata

Example 3 ($\{w\#w \mid w \in \{a,b\}^*\} \in \mathcal{L}(\det q_3\mathsf{R})$**)**

$$(q_0, ¢, a \rightarrow \epsilon, q_a)$$
$$(q_0, ¢, b \rightarrow \epsilon, q_b)$$
$$(q_0, ¢\#\$, \text{Accept})$$

$$(q_a, ¢\{a,b\}^*\#, a \rightarrow \epsilon, q_0)$$
$$(q_b, ¢\{a,b\}^*\#, b \rightarrow \epsilon, q_0)$$

Corollary 2 $\mathcal{L}(\det q\mathsf{R}) \not\subseteq \mathsf{GCSL}$ $\quad \underbrace{\mathsf{GCSL} \not\subseteq \mathcal{L}(\det q\mathsf{R})}_{\{a^{2^n} \mid n \geq 0\} \notin \mathcal{L}(q\mathsf{RRW})}$

Expressive Power $\det q$R-automata

Example 4 ($\{w\#w^R\#w \mid w \in \{a,b\}^*\} \in \mathcal{L}(\det q_3\mathsf{R})$ **)**

$$(q_0, ¢, a \to \epsilon, q_a)$$
$$(q_0, ¢, b \to \epsilon, q_b)$$
$$(q_0, ¢\#\#\$, \text{Accept})$$

$$(q_a, ¢\{a,b\}^*\#\{a,b\}^*, a\#a \to \#, q_0)$$
$$(q_b, ¢\{a,b\}^*\#\{a,b\}^*, b\#b \to \#, q_0)$$

Corollary 3 $\mathcal{L}(q\mathsf{RRW}) \not\subseteq \mathsf{GCSL}$ $\quad$ $\mathsf{GCSL} \not\subseteq \mathcal{L}(q\mathsf{RRW})$
$\mathcal{L}(\text{det-RWW}) = \mathcal{L}(\text{det-RRWW}) = \mathsf{CRL} \not\subseteq \mathcal{L}(q\mathsf{RRW})$

Expressive Power $\hat{q}\mathsf{R}$-automata

Example 5 ($(\{a^n b^n \mid n \geq 0\} \cup \{a^n b^m \mid m > 2n \geq 0\}) \in \mathcal{L}(\hat{q}_2\mathsf{R})$**)**

$$(q_0, ¢a^*, ab \longrightarrow \epsilon, q_1)$$
$$(q_0, ¢a^*, abb \longrightarrow \epsilon, q_2)$$
$$(q_0, ¢\$, \text{Accept})$$
$$(q_1, ¢a^*, ab \longrightarrow \epsilon, q_1)$$
$$(q_1, ¢\$, \text{Accept})$$
$$(q_2, ¢a^*, abb \longrightarrow \epsilon, q_2)$$
$$(q_2, ¢b^+\$, \text{Accept})$$

Corollary 4 $\mathcal{L}(\hat{q}\mathsf{R}) \not\subseteq \mathcal{L}(\mathsf{RRW})$ $\quad$ $\mathcal{L}(q\mathsf{R}) \not\subseteq \mathcal{L}(\mathsf{RRW})$